Solutions

April 9, 2025

Math 1A Worksheet #26

Name:

1. Compute the limit. Use L'Hôpital's rule when appropriate. If a more elementary method is possible, consider checking your answer in that way.

(a) $\lim_{x\to\frac{1}{2}} \frac{6x^2+5x-4}{4x^2+16x-9}$ $=$ $11/20$

(b) $\lim_{x\to 0} \frac{x^2}{1-\cos x}$ $\to \frac{0}{0}$ L'Hôpital $\to \frac{2x}{\sin(x)}$ & L'Hôpital again $\frac{2}{\cos(x)} \to 2$

(c) $\lim_{t\to 0} \frac{8^t-5^t}{t}$ $\to \ln(8/5)$

(d) $\lim_{x\to 0} \frac{\cos mx - \cos nx}{x^2}$ $= n^2 - m^2$

(e) $\lim_{x\to -\infty} x\ln\left(1-\frac{1}{x}\right)$ $= \lim_{x\to-\infty} \frac{\ln(1-\frac{1}{x})}{1/x}$ L'Hôpital $\frac{\frac{1}{1-1/x}(1/x^2)}{-1/x^2} = \frac{1}{1-1/x} \to 1$

(f) $\lim_{x\to 1^+} (\ln(x^7-1)-\ln(x^5-1))$ $= \lim_{x\to 1^+} \ln\left(\frac{x^7-1}{x^5-1}\right) = \ln\left(\lim_{x\to 1^+}\frac{x^7-1}{x^5-1}\right) = \ln(7/5)$

(g) $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{\tan^{-1} x}\right)$ $= 0$

(h) $\lim_{x\to 0^+} (\tan 2x)^x$ $\cancel{\exp\left(\lim_{x\to 0^+} \frac{\ln(\tan(2x))}{1/x}\right) = \exp\left(\lim_{x\to 0} \frac{\frac{2\sec^2(2x)}{\tan(2x)}}{-1/x^2}\right)}$

(i) $\lim_{x\to\infty} \left(1+\frac{a}{x}\right)^{bx}$

$\exp\left(\lim_{x\to\infty} \frac{\ln(1+\frac{a}{x})}{1/x}\cdot b\right) = \exp\left(\lim_{x\to\infty} \frac{\frac{b}{1+a/x}\cdot\left(+\frac{a}{x^2}\right)}{+\frac{1}{x^2}}\right) = \exp(ab)$

g.) $\frac{\arctan(x)-x}{x\arctan(x)}$ L'Hôpital $\to \frac{\frac{1}{1+x^2}-1}{\arctan(x)+\frac{x}{1+x^2}} = \frac{-x^2}{(1+x^2)\arctan(x)+x}$

$\to \frac{-2x}{1+2x\arctan(x)+1} \to 0$

h.) $\lim_{x\to 0^+} \tan(2x)^x$ $\rightsquigarrow \exp\left(\lim_{x\to 0}(x\ln(\tan 2x))\right) \to \exp\left(\lim_{x\to 0} \frac{+2\sec^2(2x)x^2}{\tan(2x)}\right)$

$\to \exp\left(\lim_{x\to 0} \frac{4\sec^2(2x)x + 2x^2(2\sec(2x))(2\tan(2x))\sec(2x)}{2\sec^2(2x)}\right)$

$= \exp\left(\lim_{x\to 0} \frac{4x+4x^2\tan(2x)}{2}\right) = \exp(0) = 1$

2. If f' is continuous, $f(2) = 0$, $f'(2) = 7$, evaluate

$$\lim_{x \to 0} \frac{f(2+3x) + f(2+5x)}{x}$$

$3f'(2) + 5f'(2)$

$= 21 + 35 = 56$

3. For what values of a, b does the equation

$$\lim_{x \to 0} \left(\frac{\sin 2x}{x^3} + a + \frac{b}{x^2} \right) = 0$$

hold?

$\sin(x) = \sum \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

$\frac{\sin(2x) - ax^3 - bx}{x^3}$

$b = 2$

$a = -4/6$

4.

(a) If f' is continuous, use L'Hôpital's rule to show that

$$\lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h} = f'(x).$$

Also sol. from limit defn.

(b) If f'' is continuous, use L'Hôpital's rule to show that

$$\lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} = f''(x).$$